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WAVE FLOW OF A VISCOUS LIQUID IN THIN LAYERS

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/All figures are appended/

I. FREE FLOW

Free Flow is a theoretical study of the flow of thin layers of a viscous liquid under the influence of a constant volumetric force and with surface tension taken into consideration. An approximate solution of the flow equation was found, which showed that the wave flow experimentally traced by a number of authors appears more stable than the laminar flow. There was obtained the form of the wave profile, phase velocity, and amplitude. The theoretical quantities, found for the critical values of Re_k at which the wave flow begins, coincide with experiments. The wave character of the flow quantitatively explains the rapid diffusion of dye along a liquid stream, as observed by Friedman and Miller.

Introduction

The flow and nature of a liquid, bounded by a hard wall and a free surface over a thin layer, up to 1-2 mm in thickness, is asserted to be basically controlled by its viscosity. The hydrodynamic problem in this case comes to be one of the simplest problems of laminar flow. It also gives the well known cubic relationship between the thickness of the layer and the discharge of the liquid per unit width of flow Q. The criterion for the stability of such a laminar flow is given by Reynold's number, which is equal to

$$Re = 4Qv^{-1} \tag{1}$$

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Where ν is the kinematic viscosity. Experimentally, the most studied case of liquid flow is the flow down a vertical wall due to gravity. Investigations by a number of authors have shown that the cubic law of laminar flow is quantitatively well substantiated by tests in the range of Reynold's numbers up to 1,500, when turbulent flow occurs. In these investigations the thickness of the layer was determined by measuring the volume of the liquid found on the wall.

Kirkbride (2) measured the thickness of the liquid layer with a micrometer and found that it appears to be greater than that obtained by the laminar rule; he explained this to be due to waves existing on the free surface.

The wave character of the flow was also uncovered by Fallah, Hunter and Nash (3). Their work showed that this wave character of motion occurs at Re 20-[illegible]. Thus it was found that this motion, which was supposed to be simple laminar, appears as a wave for almost all values of Re. To investigate the nature of this flow, Friedman and Miller (1) put a dye into the stream to calculate the maximum rate of flow. The speed of diffusion of the dye was $1\frac{1}{2}$ times faster than the flow of the surface layer of the liquid, which is greatest in laminar flow.

These investigations showed that the law of laminar flow is only part of the total effect and relates to the average thickness of the layer just as the nature of the flow is distinguished from simple laminar. The cause of this difference should obviously be sought in those hydrodynamic equations in which the force of surface tension is not taken into consideration. In the flow of liquid with low viscosity and in thin layers, even with slight deformation of the free surface, the force of surface tension assumes significant proportions, fully comparable to the forces of viscosity. It will be shown, if accounting for the forces of capillarity, that wave flow even at low velocities actually appears more stable than simple laminar flow.

The existence of a more stable wave flow has an important meaning in that it permits an explanation and description of a series of well-known physical effects, observed in the flow of thin layers, which were not very well understood up to this time. Of these effects, we will examine in this work the flow of a liquid due to the action of a gas stream over the free surface of the liquid and heat transfer in it.

Basic Relations

Besides the physical dimensions, characterising the flow, there will be density ρ, viscosity μ, and surface tension σ. For convenience, we will introduce the quantity δ (kinematic surface tension):

$$\delta = \frac{\sigma}{\rho} \qquad (2)$$

We will consider that the motion is maintained by a constant volumetric force.

The letter j in the following formulas shall designate the acceleration, due to this force, in the direction of the flow. Usually j appears as the acceleration due to gravity or to centrifugal force. The flow is two dimensional and along the X axis; the y coordinate begins at a distance a_0 from the wall (Figure 1). The variable thickness of the layer is designated by a. Velocity components at any point in the current is designated V_x and V_y. The average velocity along the X-axis in any section will be designated by V; it equals

$$V = \frac{1}{a}\int_0^a V_x \cdot dy \qquad (3)$$

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Examining the material balance of the stream for the element dx, we have

$$\frac{\partial va}{\partial x} = -\frac{\partial a}{\partial t} \qquad (4)$$

The equation of the free surface of the flowing layer is determined thus:

$$y = a_0 \cdot \phi \qquad (5)$$

The thickness of the layer therefore equals:

$$a = a_0 \cdot (1 + \phi) \qquad (6)$$

The quantity ϕ is a function of x and of time t. If it is assumed that the profile of the free surface does not change and moves with constant phase velocity k, then ϕ can be regarded as a function of one variable equal to $(x - kt)$. Then all the quantities appearing as functions of ϕ, namely $F = F(\phi)$, satisfy the following partial differential equation:

$$k \cdot \frac{\partial F}{\partial x} = -\frac{\partial F}{\partial t} \qquad (7)$$

Hereafter differentiation with respect to x will be designated by means of dots.

The average of any function of ϕ, taken with respect to length and time, is as follows:

$$\overline{f(\phi)} = \frac{1}{x_0}\int_0^{x_0} f(\phi) \cdot dx = \frac{1}{T_0}\int_0^{T_0} f(\phi) \cdot dt; \; x_0 = -k \cdot T_0 \qquad (8)$$

averaging along x will be designated by a line.

Examination of the problem shows the possibility of taking one variable, say x and placing it in an expression for ϕ when $t = 0$. We will consider the process as steady, when the thickness of the layer a has a constant mean value equal to:

$$a_0 = \bar{a} \qquad (9)$$

Consequently from expression (6)

$$\bar{\phi} = 0 \qquad (10)$$

From equations (7) and (4) we get

$$\frac{\partial}{\partial x} a(k - v) = 0 \qquad (11)$$

The difference between the velocities k and v will be designated by u; this is the velocity in the section relative to an observer who is moving with speed k of the wave front.

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Then we have

$$v = k - u \; ; \; v_0 = k - u_0 \tag{12}$$

where V_0 and u_0 designate the velocity in the middle section of the stream a_0. From (11) it follows

$$u a = u_0 a_0 = \text{const.} \tag{13}$$

The average discharge Q of the fluid according to expression (8) is:

$$Q = \frac{1}{T_0} \int_0^{T_0} v a \cdot dt = \overline{va} \tag{14}$$

Substituting the value v from (12) for steady flow, we find Q equal to

$$Q = \overline{ka} - \overline{ua} = k a_0 - u_0 a_0 = v_0 a_0 \tag{15}$$

Equations of Flow

For ordinary laminar flow of a liquid, the constant volumetric force, which is equal at every point to the viscosity force, is equal to $\mu \cdot (\partial^2 v_x / \partial y^2)$. Let us assume the following restrictions: the velocity at the wall is equal to zero; on the free surface the tangential tensions equal zero; and the velocity gradient also equals zero. By integrating this expression we obtain the well-known quadratic expression for the distribution of velocities along a section (y-coordinate):

$$v_x = 1.5 \; v \cdot \left(1 - \frac{y^2}{a^2}\right) \tag{16}$$

where V is the mean velocity in the section, determined by expression (3).

In a wave involving capillary and inertia forces, this simple distribution can be disturbed. In the examination of flow in thin layers, we shall limit ourselves to the case where wave length exceeds layer thickness. Since in this case viscosity forces play a basic role, then restrictions involving them (velocity at the wall equals zero) should be maintained. Because of this, it can be assumed that the quadratic distribution of velocity without calculating the higher terms of the series in (y/a) will be sufficiently characteristic of the flow, even for wave cycles.

Due to the small ratio of section to wave length, the influence of component V_y can also be neglected. The Naville-Stokes' equation relative to the x-axis will appear as:

$$\frac{\partial v_x}{\partial t} + \frac{1}{2} \frac{\partial v_x^2}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + g + \nu \, \nabla^2 v_x \tag{17}$$

To solve this equation, we introduce for each term its mean value along the section (y-coordinate). Thus, for quantities V_x and V_x^2 we use equation (3) and (16), respectively, and obtain:

$$\frac{1}{a} \int_0^a v_x^2 \cdot dy = \frac{6}{5} \, v^2 \tag{18}$$

Changing, in accordance with (7), the derivative with respect to time to the derivative with respect to x, we obtain:

$$\frac{\partial}{\partial x}\left(\frac{3}{5}v^{2} - kv\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + j - 3\nu\frac{v}{a^{2}} + \nu\frac{\partial^{2} v}{\partial x^{2}} \qquad (19)$$

We are looking for the approximate value of the function ϕ that satisfies this equation and possesses a steady-state periodic character. We assume that in expression (5) the amplitude of oscillation of the wave surface will always be less than the average thickness of the layer a_0. From this it follows that the amplitude $|\phi|$ is less than unity:

$$|\phi| < 1 \qquad (20)$$

We will evaluate the order of magnitude from the exponents of ϕ occurring in terms with the same coefficients. We also assume that the derivatives of ϕ are determined by the following relations.

$$|\phi|^{2} > a_0 |\dot{\phi}|; \; |\phi|^{3} \geq a_0^{2} |\ddot{\phi}| \text{ etc.} \qquad (21)$$

In our examinations of liquid flow, the pressure gradient in equation (19) can be set up in two ways: (1) pressure of a gas current on the liquid's free surface; and (2) forces of surface tension.

First, we shall examine the flow in the absence of a gas current. According to well known expressions for pressure gradients produced by capillary forces it will equal

$$\frac{\partial p}{\partial x} = -\sigma \cdot \frac{\partial}{\partial x}\frac{\ddot{a}}{(1+\dot{a}^{2})^{3/2}} \qquad (22)$$

According to condition (21), the denominator can be considered as equal to unity, correct to the second term of its expansion.

__First Approximation__

We will introduce into equation (19) the function ϕ. According to (6), (12) and (13), the velocity v and its derivative with respect to x are:

$$v = k - \frac{u_0}{1+\phi}; \qquad \dot{v} = \frac{u_0}{(1+\phi)^{2}}\,\dot{\phi} \qquad (23)$$

We will introduce the symbol z for the ratio of phase velocity k to velocity v_0:

$$z = \frac{k}{v_0} \qquad (24)$$

If all coefficients retain only high-order terms, then equation (19) yeilds the following:

$$\delta a_0 \dddot{\phi} + v_0^{2}(z-1)(z-1.2)\dot{\phi} + 3\left(j - z\frac{\nu v_0}{a_0^{2}}\right)\phi + \left(j - 3\frac{\nu v_0}{a_0^{2}}\right) = 0 \qquad (25)$$

When $\phi = 0$, we have simple laminar flow. Designating the thickness of the layer of the current by m and the mean velocity by W and introducing the symbol Q from (15), we get from this expression the general relationship

$$j = 3\nu W m^{-2} = 3\nu Q m^{-3} \tag{I}$$

When ϕ is not equal to zero, we get a linear differential equation of the third order. In order that it may have a steady-state periodical solution, the constant term and the coefficient of ϕ must equal zero. It follows from this that the relation in (I) should be maintained, and therefore, in the first approximation in wave flow, the thickness of the layer a_0 will approximate the thickness m which appears in usual laminar flow. The velocity V_0 in the average section also equals the velocity W in the first approximation.

Setting the coefficient of ϕ equal to zero gives:

$$j = z\nu V_0 a_0^{-2} \tag{II}$$

This equation is important for the existence of steady-state periodic flow. Substituting in (II) the value of j from (I), we see that in all cases z should be positive and in the first approximation $z \approx 3$. Consequently, the phase velocity k along the direction of flow is three times the average velocity V_0 in the section of flow.

If the length of the waves is designated by λ and there is introduced the symbol:

$$n = \frac{2\pi}{\lambda} \tag{III}$$

then the periodic solution sought for is:

$$\phi = \alpha \cdot \sin nx \tag{26}$$

The quantity α is called the amplitude. The quantity n is determined from:

$$n^2 = (z-1)(z-1.2)\, V_0^2 (\delta a_0)^{-1} \tag{IV}$$

For given z and a_0, this expression establishes the length of the waves. The quantity V_0 is determined from (15) by the discharge Q. Later, Q will be made independent variable by means of the functions of which we will study the flow.

Second Approximation

To find the degree of accuracy of the first approximation and the restrictions under which wave flow can take place, second approximation is necessary. The second approximation is obtained by introducing into equation (25) terms with ϕ of the second order of magnitude. Having done this, we get the following nonlinear differential equation:

$$\delta \cdot a_0 \cdot \ddot{\phi} \cdot (1+3\phi) + V_0^2 \cdot (z-1)(z-1.2) \cdot \left(1 - \frac{z\cdot\phi}{5z-6}\right) \cdot \dot{\phi} +$$
$$3j\phi^2 + (j - z \cdot \frac{\nu \cdot V_0}{a_0^2})\,\phi + (j - 3\frac{\nu V_0}{a_0^2}) = 0 \tag{27}$$

(Note: ν Greek "nu")

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We are now looking for a stable steady-state periodic solution of this equation after adding to solution (26) some terms of double periodicity. The terms of the second order of magnitude may similarly be set equal to zero, after obtaining the following:

$$\phi = \alpha \cdot \sin nx + 0.28\,\alpha^2 \cos 2nx - \frac{j\alpha^2}{4n^3 a_0 \varepsilon} \cdot \sin 2nx + \cdots$$

In order that this solution may result, the earlier conditon (II) of the equation must maintain the null value of the coefficients of ϕ. Equating to zero the constant terms which are independent of ϕ and substituting (V) in equation (27) gives:

$$j\left(1 + \frac{3}{2}\alpha^2\right) = 3\nu V_0 a_0^{-2} \qquad (28)$$

This equation shows that in the second approximation the average thickness a_0 of the layer is actually related to amplitude α. We will designate by β the ratio between average thickness a_0 of layer and thickness m of layer in a laminar flow with discharge Q:

$$a_0 = \beta \cdot m \qquad (29)$$

Then from expressions (28) and (I), we obtain:

$$\beta^{-3} = 1 + \frac{3}{2} \cdot \alpha^2 \qquad (30)$$

It is clear from this expression that β depends on the amplitude and will always be less than unity and that consequently in a wave flow the average thickness of the layer will be less than in a laminar flow. As will be clear what follows, this makes the steady-state wave flow more stable than the laminar flow. Expression (IV) determining n, i.e., the wave length, remains unchanged in the second approximation.

Expression (V) shows that ϕ possesses a second-harmonic term $\cos 2nx$ due to lack of symmetry in the conditions of liquid flow. When the quantity a becomes less than its average value a_0, then the conditions of flow will be distinguished from those when a has a value greater than a_0. The coefficient of $\cos 2nx$ is small; this shows that even at significant amplitudes the simple sine form of the waves will not essentially be altered.

The term with $\sin 2nx$ arises in a wave flow because of the absence of equilibrium between viscosity and volumetric forces. In maintaining condition (II), this absence of equilibrium will only cause the formation of harmonics of the second or higher order. The size of the coefficient of these terms rapidly decreases with increase in n, with shortening of the wave length, since n enters into the coefficient as of the third degree. A noticeable influence on the wave pattern will be shown by this term only when wave flow arises. Because of this, the influence on wave pattern by a term with $\sin 2nx$ can be disregarded practically throughout the range of wave flow. This term is necessary to create the critical conditons for the occurence of a wave flow. Thus, the second approximation shows that the sine form represents well the pattern of the wave surface. It may be presumed that terms of the next order will have even less influence, but their calculation is awkward.

If in expression (V) time is introduced, then the periodic solution in the second approximation for a will be:

$$a = a_0\left[1 + \alpha \cdot \sin n(x - kt) + 0.28\,\alpha^2 \cos 2n(x - kt) - \frac{j\alpha^2}{4a_0\varepsilon n^3} \cdot \sin 2n(x - kt) + \cdots\right] \qquad \text{(Va)}$$

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Determination of the Amplitude

The amplitude α can be determined from the stability conditions of wave flow and energy balance. In the examination of flow conditions, energy dissipation will be found to occur only as a result of viscosity forces. We will designate by dE_μ the amount of energy dissipation in an element of volume consisting of an entire cross section of flow and in element length dx; then in accordance with a well-known hydrodynamic formula [4] and with certain restrictions, it will equal

$$dE_\mu = -dx \cdot \mu \int_0^a \left(\frac{\partial v_x}{\partial y}\right)^2 \cdot dy \tag{31}$$

Substituting the value v_x from (16) and integrating, we obtain the dissipation of energy in the element of length dx:

$$dE_\mu = -3\mu \cdot \frac{v^2}{a} \cdot dx$$

The average dissipation per unit length equals:

$$\overline{E}_\mu = -3\mu \, \overline{v^2 a^{-1}} \tag{32}$$

Substituting the vales of v and a from expressions (6) and (23) and the value of z from (24) and designating the function by F, we then have:

$$F = \frac{1}{\lambda} \int_0^\lambda \frac{(1+z\phi)^2}{(1+\phi)^3} \cdot dx \tag{33}$$

we find that for discharge Q the average dissipated energy per unit length equals

$$\overline{E}_\mu = -3\mu \cdot Q^2 a_0^{-3} \cdot F \tag{34}$$

If $\phi = 0$, i.e., if wave flow is absent, then $F = 1$ and the dissipated energy equals that in the usual laminar flow.

Dissipation energy occurs only because of the work of volumetric forces. The average work per unit length for discharge Q equals

$$\overline{E}_j = j\rho \cdot \overline{va} = j\rho Q \tag{35}$$

For a given discharge Q, this work is constant; consequently the amount of energy E_μ dissipated during a steady-state cycle is constant. Comparing this and the previous expression, we get for the average thickness of the layer

$$a_0^3 = 3\nu \, Q \cdot j^{-1} \cdot F \tag{36}$$

It is clear from this expression that the smaller the value of F, the smaller the average thickness a_0 of the layer.

Let us compute the quantity F. We will assume that a definite integral of the following form is known:

$$\frac{1}{\lambda} \int_0^\lambda \frac{dx}{b + c\phi} = f(c, b) \tag{37}$$

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Differentiating f with respect to the parameter c or b and then setting c and b equal to unity, we can then obtain all of the three types of integrals into which expression (33) breaks down, if, for example, the parentheses of the numerator are removed, thus:

$$\frac{1}{2}\frac{\partial^2 f}{\partial b^2}\Big|_{\substack{b=1\\c=1}} = \frac{1}{\lambda}\int_0^{\lambda}\frac{dx}{(1+\phi)^3} \tag{38}$$

etc.

Regarding only the steady-state periodical flow, let us introduce the function ϕ from (V) into expression (37) and limit ourselves at first to terms with the first harmonic. Then we will obtain the well-known integral:

$$\frac{1}{\lambda}\int_0^{\lambda}\frac{dx}{b+c\alpha\cdot\sin nx} = (b^2-c^2\alpha^2)^{-\frac{1}{2}} \tag{39}$$

Carrying out the indicated operations, we get for F the expression:

$$F = \frac{1}{2}\left\{2+\alpha^2\left[1-6z+z^2(1+2\alpha^2)\right]\right\}\cdot(1-\alpha^2)^{-\frac{5}{2}} \tag{40}$$

The curve for z versus α^2 when $F=1$ is shown in Figure 2.

It is clear from the drawing that this curve and the z -axis is the maximum closed area and that points lying within this area corresponding to values of z and α^2 when $F < 1$.

To find the minimum value of F, we construct the curves when $\partial F/\partial(\alpha^2)=0$ and $\partial F/\partial z = 0$ (Figure 2). The intersection of these curves gives the point M with coordinates $z = 1.5$ and $\alpha^2 = 0.5$; there $F \cong 0.7$, which is the lowest possible value of this function.

Let us suppose that in a liquid flow of ordinary laminar nature there occurs a fluctuating sinusoidal disturbance of such a character that its phase velocity will have a value falling in the area defined by the curve for $F = 1$. Then the dissipated energy E_μ will be smaller than the energy E_i transmitted to the flow. Thus there begins a growth of kinetic and capillary energies which leads to an increase in the amplitude of oscillation. This process will last until F reaches the allowed minimum value and the thickness of the flow contracts to a value determined by expression (36).

Since, in this work, we are limited to steady-state periodic solutions, then in order that F may have the minimum value, the value of z must satisfy condition (II) for equating the coefficient of ϕ in basic equation (25) to zero.

Substituting the value j from (I) and a_0 from (29) in expression (II), we obtain:

$$z = 3\cdot\beta^3 \tag{41}$$

If we compare expression (36) in a wave flow with general laminar flow (I) and introduce the ratio of thickness a_0 to m from (29), then we obtain:

$$\beta^3 = F = \frac{1}{3}\cdot z \tag{42}$$

This equation, together with the condition for the minimum value F, determines the quantities α^2 and z. To solve this problem, we calculate the value of F_m corresponding to the curve $\partial F/\partial(\alpha^2) = 0$. We plot the curve $z = 3F_m$

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and find the point of its intersection with the curve $\partial F/\partial(\alpha^2) = 0$. It is indicated in Figure 2 as 0 and gives the desired values of the quantities z and α^2. If we round off the calculated numbers, within the limits of accuracy of a slide ruler, then we obtain.

$$\alpha^2 = 0.21 \text{ or } \alpha = 0.46;\ z = 2.4;\ F = \beta^3 = 0.8 \text{ or } \beta = 0.93 \quad \text{(VI)}$$

Thus, in accordance with expression (29), it is seen that in steady-state periodic wave flow with equal discharge rates Q, the average thickness a_0 of the current during wave flow will be approximately 7 percent less than m for laminar flow. From the obtained value of α it is clear that the amplitude of waves for all values of the discharge rates Q will have a certain size, equal to 0.46 of the average thickness a_0 of the flow. The phase velocity will be equal to 2.4 of the velocity V_0 in the average section, i.e., it considerably exceeds V_0.

Having the first approximation for α^2, we can evaluate the error brought about by omitting, during the calculation of integrals (39), the second-harmonic terms of expression (V) for ϕ. It can be shown that this error is of the order:

$$(0.28\alpha)^2 = 0.018 \quad (43)$$

i.e., less than 2 percent.

Let us assume that the length of the wall along which the liquid flow from its point of supply is equal to x_0 and is sufficiently large so that the flow over a great part of the length is steady. Then for a given discharge rate Q, the potential energy of this layer will be equal to

$$\frac{1}{2} a_0 x_0^2 j \rho \quad (44)$$

With complete equilibrium of all remaining forms of energy and with steady-state flow, the most stable will be that type of flow for which quantity (44) has the least value and consequently for which the mean thickness a_0 of the layer will be a minimum. Thus, for a given discharge rate Q, since a_0 will be smaller than m, the studied wave flow will be more stable than the laminar flow.

It must be noted that our analysis even within the limits of accepted approximations does not permit ascertaining whether the steady-state sinusoidal wave pattern [expression (V)] obtained from equation (27) is the most stable type of wave flow. A more complete study of this type of flow will probably lead to the finding of wave flows which occur at even smaller average thicknesses of the layer and which consequently are even more stable. It can be seen that these types of flow will appear as variations of the obtained sinusoidal flow in which the waves will appear bunched and the amplitude of the waves somewhat larger and the phase velocity somewhat lower. In the first steps to studying this type of flow, the period solution, as was obtained by experiment, can serve as a good approximation for describing those physical effects discussed in the introduction.

Maximum Values in Wave Flow

Substituting in expression (IV) the values obtained for z and F from (VI), introducing discharge rate Q and using expression (36), we obtain for n^2:

$$n^2 = (z-1)(z-1.2)\frac{1}{3F}\cdot\frac{jQ}{\delta\nu} = 0.7\frac{jQ}{\nu\delta} \quad (45)$$

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From expression (III) we get the wave length equal to

$$\lambda = 7.5\left(\frac{\nu \delta}{Q j}\right)^{\frac{1}{2}} \qquad \text{(VII)}$$

The average thickness of the layer is determined from expression (36) by substitution of the value F = 0.8:

$$a_0 = 1.34\left(\frac{\nu Q}{j}\right)^{\frac{1}{3}} \qquad \text{(VIII)}$$

These two expressions permit determining that range of Re and discharge rate Q allowed due to the nature of our solution. The extremal values of the derivatives (21) determine the wave length limits which approximation actually permit us. Putting the values of ϕ and α in (21) we get:

$$\frac{\lambda}{a_0} > \frac{\lambda_{st}}{a_0} = \frac{2\pi}{\alpha} = 13.7 \ \text{[NOTE: st: straight]} \qquad (46)$$

This expression determines the least wave length at which α and $\varkappa$ can be considered to depend only slightly on the wave length; the shorter the wave, the less dependable the adopted approximation. Of course, this arrangement does not limit the existence of the wave flow and at smaller wave lengths, as was shown, it can be seen that a decrease in wave length causes α to grow and $\varkappa$ to decrease. Condition (46) also agrees with our other demands that wave length should be significantly larger than thickness of the stream.

The maximum wave length at which wave flow begins cannot exactly be solved. It would be most natural to determine those values of the coefficients containing the quantity n in our expression (V) which permit a complete solution. Having only the first two (sic) terms of this analysis, we can only assume that the convergence of the series will be assured when the coefficient of $\alpha^4 \sin 2nx$ is less than unity; that is:

$$j/4n^3 a_0 \delta \leqslant 1 \qquad (47)$$

This inequality determines the least value of n.

The maximum value at which wave flow occurs can be ascertained from the following more physical considerations: Let us find the mean potential energy of surface tension per unit length of flow. It equals the surface area of the waves multiplied by σ :

$$\overline{E}_\sigma = \sigma \cdot \overline{[1 + (a_0 \phi')^2]^{\frac{1}{2}}} \qquad (48)$$

Substituting the value of ϕ from (V) and keeping only the largest terms in n, we then obtain:

$$\overline{E}_\sigma = \frac{1}{4}\sigma a_0^2 \alpha^2 n^2 \cdot \left[1 + 4\left(\frac{j\alpha}{4a_0\delta}\right)^2 \cdot n^{-6} + \dots\right] \qquad (49)$$

It is clear that increase in λ and, consequently decrease in n, as long as the quantity in the parentheses is small, causes the surface energy $\overline{E}_\sigma$ also to decrease. But when the term in the parentheses becomes large, as for a certain wave length λ_k a change of sign occurs in the derivative $\partial \overline{E}_\sigma / \partial n$ and the energy begins to increase. Physically this corresponds to the appearance in the regions of the wave peaks of a noticeable reverse bend tending to break them up. Such wave patterns can be assumed to be unstable; thus maximum wave length is obtained from the equation:

$$\frac{\partial E_\sigma}{\partial n} = 0 \qquad (50)$$

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Disregarding the variation of a_0 with wave length, from (42) we get

$$j/4n^3 a_0 \delta \leqslant 0.77 \qquad (51)$$

Since n involves the cube root of the right quantity, both expressions (47) and (51) give approximate values for the critical wave length. Taking inequality (51) and introducing the value of n from expression (45), the maximum ratio of λ_k to a_0 becomes

$$\frac{\lambda}{a_0} \leqslant \frac{\lambda_k}{a_0} = 13.5\, V^{-1}\, Q_k \qquad (52)$$

If we introduce the value of the Re number from (1), we obtain for the limit of wave flow, upon substituting values from (VII) and (VIII):

$$Re_k = 2.43 \left(\frac{\delta^3}{j\nu 4}\right)^{1/11} = 0.3 \frac{\lambda k}{a_0} \qquad \text{(IX)}$$

The critical value of the Re number thus obtained, as will be clear from the data given, establishes definitely the transition point from laminar flow to wave flow.

Description of Wave Flow

The results obtained make it possible to describe periodic wave flow of a liquid layer. Figure 3 shows a stream section calculated for a certain moment of time on the basis of expression (V). The scales of a_0 and λ were so selected that the diagram would be of convenient size. Actually the ratio of wave length to section is considerably greater than that shown in the drawing. The curve relates to the case where the coefficient of sin 2nx is zero. Since this term involves n^{-3}, its size after wave formation will rapidly decrease with increase of discharge rate Q; the wave pattern approaches the described curve. Figure 3 marks the average thickness a_0 of the layer and also the thickness which would have been present for ordinary laminar flow.

The average velocity in any section of the current is determined according to expressions (12), (23) and (24):

$$V = V_0 \cdot \left(z - \frac{z-1}{1+\phi}\right) \qquad (53)$$

By substituting the values of z and ϕ, the velocity can be calculated in any section. The maximum velocity V_m is obtained in the widest part of the current. In the narrowest part, the mean velocity V_a is not only low, but is also negative in direction. The mean velocity $\bar{V}$ along the length of the stream can be computed from expression (53) and integral (39). The quantities calculated for the wave pattern in Figure 3 are:

$$V_m = 1.44\, V_0;\ V_a = -0.2\, V_0;\ \bar{V} = 0.83\, V_0 \qquad (54)$$

In a certain section of the wave, the velocity equals zero. The position of this section is determined from expression (53) by equating to zero. In Figure 3 the sections where velocity is zero are marked by the lines OO.

In every section the distribution of the velocity along the y-axis, according to expression (17), is a quadratic function in y. Thus on the surface of the current the velocity will have the greatest value and is 1.5 times the mean. In Figure 3, the velocity is schematically indicated by arrows at various points. In examining the picture obtained of the velocity distribution, the flow can clearly characterized not as a wave, but as elongated drops of liquid rolling along the wall.

Thus analysis shows that, beyond a certain thickness of layer, attained at the critical Re value Re_k (IX), the more stable flow corresponds to the breaking down of the stream into separate drops. As was already mentioned, although the period solution found is more stable than the laminar one, a more complete and exact analysis may uncover the possibility of an even less regular structure for the wave surface. Thus the dimensions and distribution of the drops may not always be regular, as presented in Figure 3, although the average values of amplitude and wave length and phase velocity should not vary too much from those obtained. The tendency to gather into drops in the stream may also occur not only along the x-axis lengthwise with the direction of flow, but also along the y-axis perpendicularly. The problem here ceases to be two-dimensional, and its solution encounters further mathematical difficulties. Experimental study of wave flow can significantly aid in further analysis of this complex type of flow.

Comparison with Experimental Data

In Figure 4 are included the ratios λ/a_0 calculated from expressions (VII) and (VIII) against various values of the Re [1] number, for water and tuolene and liquid air; the flow occurs along a vertical wall under the influence of gravity ($g = 981$). The accepted physical constants are given in the following table:

	ρ	μ	σ	Re_K	Re_{st}
Water	1	$0.9 \cdot 10^{-2}$	71	23	200
Tuolene	0.86	$0.55 \cdot 10^{-2}$	29	21	170
Liquid air	0.9	$0.174 \cdot 10^{-2}$	9.4	24	215

In Figure 4 the straight line is determined by equation (IX); its points of intersection with the curve give the values of the critical Re_K and λ_K/a_0. The quantities obtained are placed in the table. As can be seen, the Re_K values lie close to the experimental ones [1], 25 - 5. In the drawing, there is also a horizontal straight line given by equation (46); it intersects the curves at points corresponding to λ_{st}/a_0 up to which (points) the allowances made in the calculations cited may be considered as realized. Re_{st} corresponding to these values is also indicated in the table.

The accuracy of the data for the thickness of the stream cited by various authors does not permit a reliable check of the 7 percent difference between the thickness of the layer, calculated for laminar flow (I) and for flow (VIII).

During a wave cycle the liquid found on the outer surface will move with the greatest velocity, which will always be less than the phase velocity. The liquid particles therefore, found in the outer surfaces will, consequently, be found in succession in various parts of the wave profile and their velocity will not only change in quantity, but also, as was shown, even in sign. The mean velocity of the particles on the outer surface equals $1.5\,\overline{V}$ according to expression (16) and $1.25\,V_0$ according to the data in (54). The velocity will obviously be less, in comparison with laminar flow, than 1.5w, since velocity v_0 is only 7 percent greater than w for the same discharge rate Q.

At the beginning of this article, it was indicated that Friedman and Miller [1] observed a diffusion rate of dye in a liquid equal to a velocity 2.4w. Thus, this high value cannot be explained simply by an increase of the velocity of particle motion of a liquid on the surface during wave flow. Such rapid diffusion of dye in the stream is actually conditioned by the wave character of the flow, but its origin is somewhat different. To make its mechanism clear we will examine the model in Figure 5.

Let us assume that the narrow parallel gap between Plates 1 and 2 is filled with liquid and on the left, the shaded part, the liquid is heavily tinted up to the limit designated by line cd (Figure 5A).

Now to the upper Plate 1 there is applied a horizontal oscillatory motion with amplitude b and period T. Then in a period $\frac{1}{2}$T the upper plate will take the position described in Figure 5 B and the limit of the tinted part cd will now be inclined to the right. In the time of movement $\frac{1}{2}$T, the dye will diffuse due to the thinness of the layer through the layer and penetrate into the area below, and the border of the colored area will extend to the line c unity, d. In the time T, the upper Plate will return to its original position (Figure 5 B), but from the diffusion through the thin layer, the fluid will remain c d. During succeeding oscillation, the process will be repeated (Figure 5, C), but now the limit c d of the diffusion of dye will be found in the expanse b from the left end of the plate and will expand to 2b. After completion of the second oscillation (Figure 5 D), the coloring of the liquid in the gap will first be intensified in area and secondly, the limit will be extended to the area 2b. The process will continue in this manner and although the liquid on the average will not have a longitudinal flow all the dye will be found to diffuse with an average velocity of b/T. In such a process, of course, the intensity of the color along the path of diffusion will not be equal and will always decrease toward the edge.

It can be considered that a similar effect occurs in the diffusion of a dye along a thin layer of flowing liquid during wave flow. Here the color limit will move with the maximum velocity in the stream. According to (53), it is equal to $V_m = 1.5 \cdot V_m = 2.1$ v or 2.3w of the mean in laminar flow, if it were possible for the laminar to take place at the same discharge rate Q.

If the velocity on the surface of the liquid at its peak during laminar flow is $W = 1.5w$, then from the expression for discharge (15) we have

$$W = 1.5\frac{Q}{m} \tag{55}$$

If we substitute the value of m from (I) and let $g = 981$, we then obtain for laminar flow:

$$(W\nu)^3 = 1100(Q\nu)^2 \tag{56}$$

Similarly we get for wave flow:

$$V_m = 2.1\ Q \cdot a_0^{-1} \qquad (\ldots)$$

Substituting the value of a_0 from (VIII) we get

$$(V_m\nu)^3 = 3800(Q\nu)^2 \tag{58}$$

If the quantities within parentheses in (56) and (56) are taken as coordinates, then we get two curves. In the laminar area, the diffusion of dye will follow the curve for (56) and beyond the value Q_k (52), at which wave flow begins, it will follow the curve for (58). The curve thus obtained is shown in Figure 6. Friedman and Miller graphically presented the results of their experiments with water. The points obtained by them for the velocity of diffusion of dye are copied in Figure 6.

Considering the difficulty of visual observation of the diffuse edge of the dye which obviously occasions the great dispersion of the points in Figure 6, the conformity of the experimental data with the calculated curve can be considered satisfactory. The correctness of the presented mechanism of the process of dye diffusion is substantiated also by the fact that, in the experiments cited, the authors point out that the edge of dispersion of the dye was greatly diffused.

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[Appended figures follow.]

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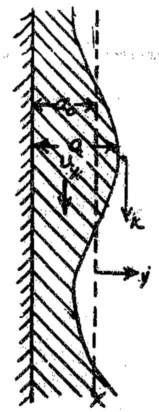
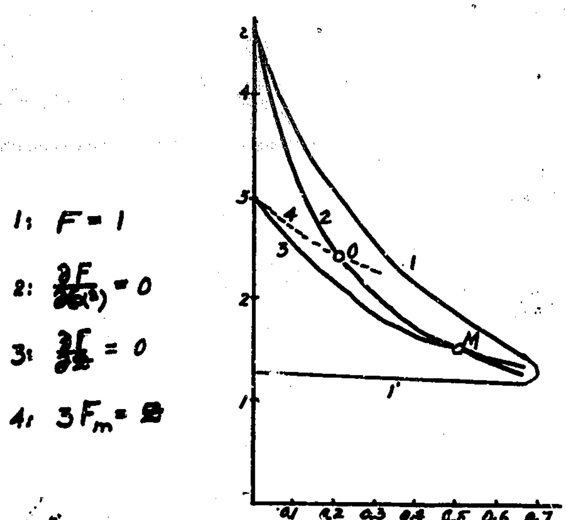


Figure 1



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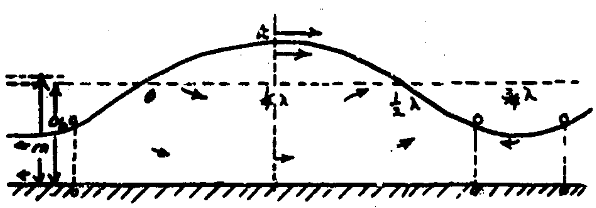

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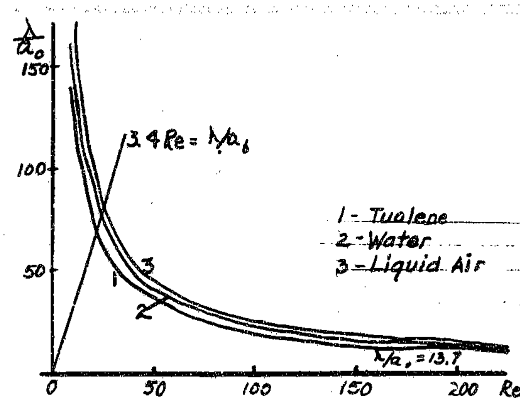

Figure 4

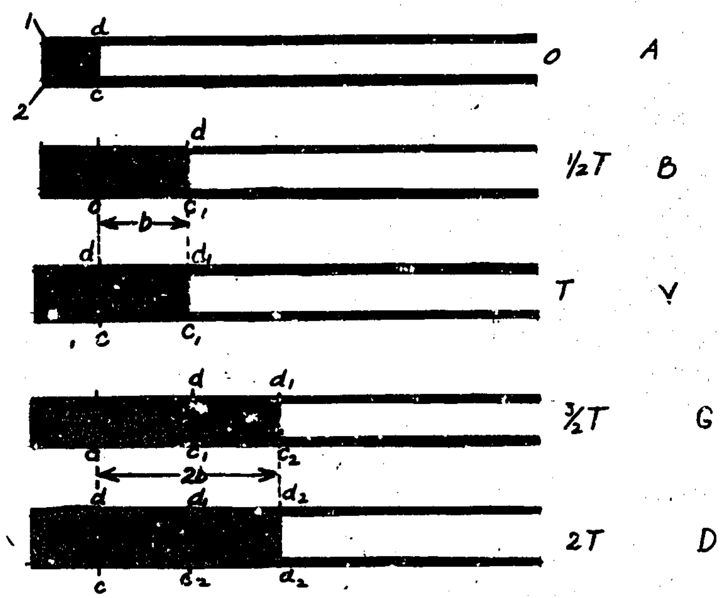

Figure 5

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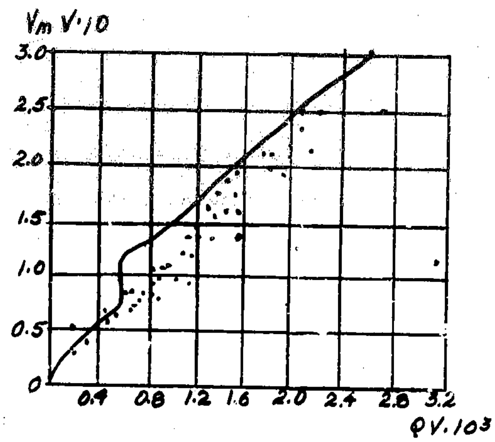

Figure 6

- END -